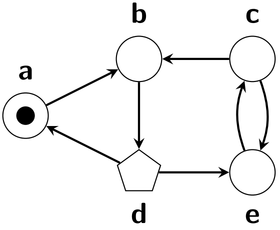
Attracting Tangles to Solve Parity Games

Tom van Dijk (JKU Linz)

CAV, 17 July 2018

Parity Games

- A parity game is played on a directed graph
- Two players: Even ○ and Odd ⬠
- The players move a token along the edges of the graph
- Each vertex is owned by one player who chooses a successor



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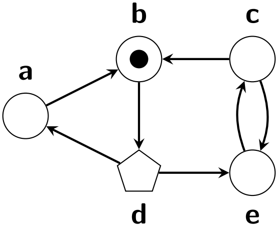

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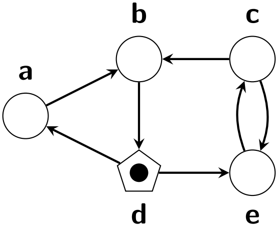

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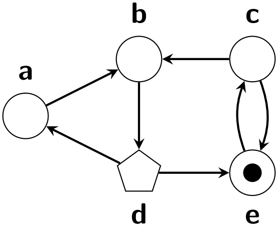

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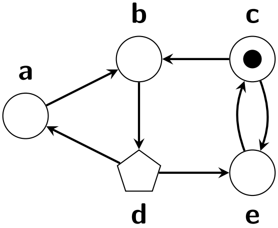

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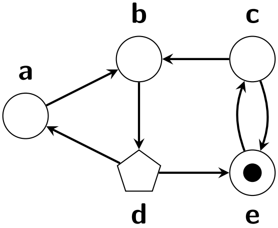

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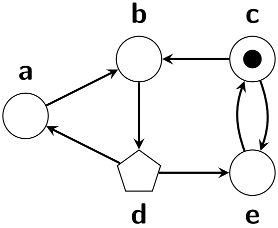

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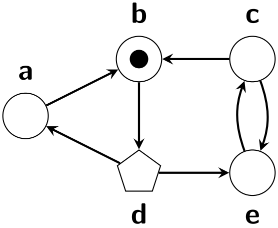

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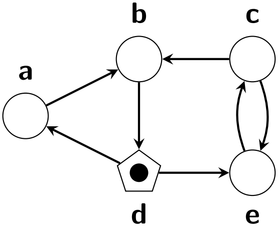

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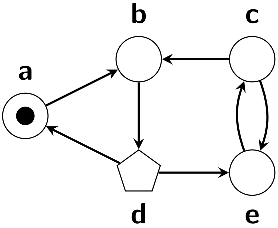

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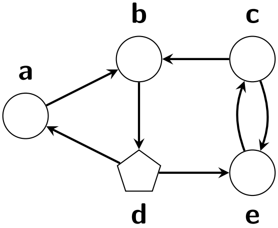

How do we determine who wins a play?

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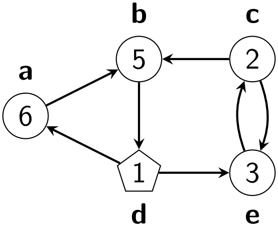

- Each vertex has a priority $\{0, 1, 2, \ldots, d\}$
- Highest priority seen infinitely often determines winner
- Player Even wins if this number is even

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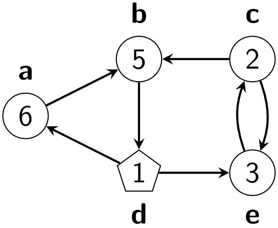

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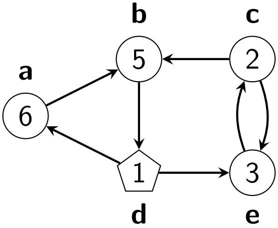

A player wins a vertex if it has a strategy
to win all plays from that vertex

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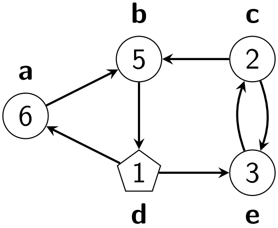

Which vertices are won by which player?

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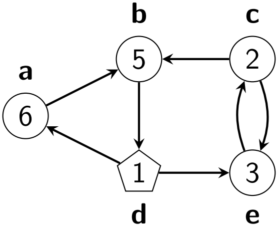

Player Odd wins all vertices with strategy $\{\,\mathbf{d} \to \mathbf{e}\,\}$

Parity Games

Known facts of parity games

- Some vertices are won by Even, some vertices are won by Odd
- The winner has a memoryless strategy to win

Memoryless strategy

"If I always play from v to w, then I win all plays from v"

Parity Games

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Memoryless strategy

"If I always play from v to w, then I win all plays from v"

Solving a parity game

- Determine the winner of each vertex
- Compute the strategy for each player

Parity Games

Why do we want to solve parity games?

- As expressive as nested least and greatest fixpoint operators
- Polynomial-time equivalent to:
  - modal μ-calculus model-checking
  - solving Boolean Equation Systems
- Backend for LTL model checking and LTL synthesis

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Open question: Is solving parity games in **P**?

- The problem is in $\mathbf{NP} \cap \mathbf{co\text{-}NP}$
- The problem is in $\mathbf{UP} \cap \mathbf{co\text{-}UP}$
- It is believed a polynomial solution exists

Parity Games

(Incomplete list of) published algorithms

McNaughton/Zielonka	$\mathcal{O}(e \cdot n^d)$, $\mathcal{O}(2^n)$	1998
Small Progress Measures	$\mathcal{O}(d \cdot e \cdot (n/d)^{d/2})$	1998
Strategy Improvement	$\mathcal{O}(n \cdot e \cdot 2^e)$	2000
Dominion Decomposition	$\mathcal{O}(n^{\sqrt{n}})$	2006
Big Step	$\mathcal{O}(e \cdot n^{d/3})$	2007
APT	$\mathcal{O}(n^d)$	2016
Priority Promotion	Exponential	2016
Quasi-Polynomial (multiple)	$\mathcal{O}(n^{6+\log d})$	2016 – 2018
Tangle Learning	(tbd)	2018

Parity Games

Attractor computation

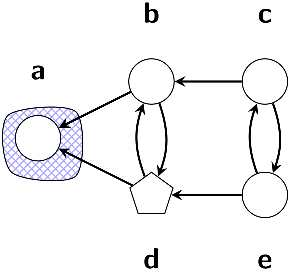
Compute all vertices from which player α can ensure arrival in a target set

Start with the target set A, then iteratively add vertices to A:

- All vertices of α with an edge to A
- All vertices of $\overline{\alpha}$ with only edges to A

Example of attractor computation

Computing the ⬠-attractor to **a**



Initial set: $\{\,\mathbf{a}\,\}$
Can attract: **d** but not **b**

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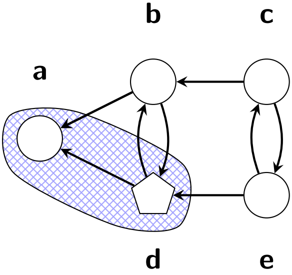

Current set: $\{\mathbf{a}, \mathbf{d}\}$
Can attract: **b** but not **e**

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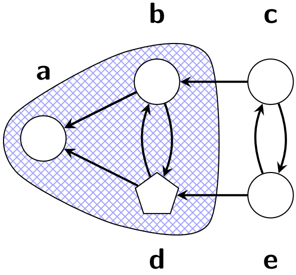
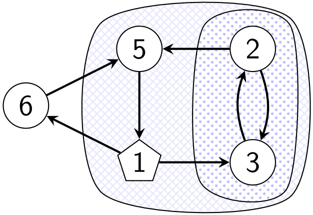


Current set: $\{\mathbf{a}, \mathbf{b}, \mathbf{d}\}$
Can attract: neither **c** nor **e**

Contributions

The notion of a tangle

- A tangle is a strongly connected subgraph, such that all plays that stay in the tangle are won by one player
- Therefore the other player must leave the subgraph



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The role of tangles in parity game solving algorithms

- Many algorithms implicitly explore tangles
- They often explore the same tangles over and over again
- This leads to an exponential number of steps

Contributions

Main contribution

Tangles can be used with attractor computation!
The loser **must leave** the tangle
Thus we can attract vertices of a tangle together

Contributions

Tangle learning

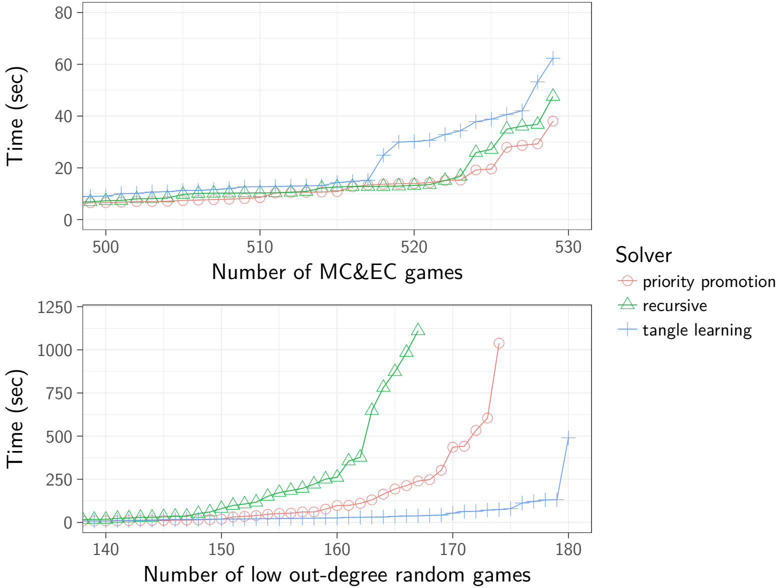
- Extend attractor computation to attract tangles
- Use extended attractor computation to decompose the game
  - Compute attractor set to highest priority
  - Remove this attractor set from the game
- Analyse decomposition to compute new tangles
- Refine decomposition with new tangles
- Repeat this until the game is solved

Empirical evaluation

- Evaluated using Oink (TACAS 2018)
- Benchmarks
  - Model checking and equivalence checking games [Keiren 2015]
  - Random games
  - Random games with max out-degree 2
- Runtimes in seconds, timeout 20 minutes

Solver	**MC&EC**	**Random**	**Random (low degree)**	
	time	time	time	timeouts
priority promotion	503	21	12770	6
recursive	576	21	23119	13
tangle learning	808	21	2281	0

Empirical evaluation



Conclusions

- Fast moving field with great progress in the last few years
- Existing algorithms implicitly explore tangles repeatedly
- Tangles can be used with attractor computation
- Tangle learning explicitly remembers tangles
- See for yourself: `https://www.github.com/trolando/oink`

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